Hounslow Education Partnership

KING'S MATHS SCHOOL

# Start as they need to go on

## Developing algebra skills for A level success

Please

- change your Zoom name to "Full name, School"
- keep your video on but microphone off
- Have a paper and pencil, or something more modern!, to hand
- Tell us in the Chat: "my favourite BBQ dish is ..."

# My objectives

To suggest

- topics
- examples
- rationale

to support and inform your construction of a "Core skills in Algebra" development / consolidation programme for Y12 A-level mathematicians that will prepare them well for Y13 success.

# Trouble ahead!

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1. What are the coordinates of the turning points of the curve $y = \frac{x^2+1}{\sqrt{x^4+1}}$ ?

2. What is the general solution of $\frac{dy}{dx} = \frac{y}{x}$ ?

3. A particle of mass 5kg is suspended by two light strings at angles of $40°$ and $70°$ to the horizontal, and also a force 20N acts horizontally. Work out the tension in each string.

# ALL students must

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**REASON MATHEMATICALLY** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

# ALL students must

become **FLUENT** in the fundamentals of mathematics, including through **varied and frequent practice with increasingly complex problems** over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

[illegible] justification or proof using mathematical language.

# ALL students must

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become **FLUENT** in the
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using mathematical
language.

**SOLVE PROBLEMS** by applying their mathematics to a **variety of routine and non-routine problems** with increasing sophistication, including breaking down problems into a series of simpler steps and **persevering in seeking solutions**.

# Routine and non-routine

Today

- Factorising
- Simultaneous equations
- Indices & logarithms
- Algebraic fractions
- "Algebra-tisation"

Next week

- Inequalities
- Completing the square
- Etc.

# In conclusion (now?)

# In conclusion

- “Unfinished business” leads to “trouble ahead”
- General tactics & Specific strategies
- Desirable difficulty, productive struggle
- Get the maths from them
- Accommodate don’t assimilate

# Factorising

"Factorise $x^2 + 5x - 14$" at GCSE

# Factorising

"Factorise $x^3 + 3x - 14$" at A level

# Factorising

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- Factorise $x^3 - 19x + 30$

- Factorise $x^3 - 6x - 9$

- What is the remainder when $x^4 + 3x^3 + 5$ is divided by $x + 2$?

# Factorising

- What is the remainder when $x^4 + 3x^3 + 5$ is divided by $x + 2$?

# Factorising

Simplify

- $81(x+y)^2 - 16(x-y)^2$

- $(x-3+\sqrt{2})(x-3-\sqrt{2})$

# Factorising

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Factorise

- $4x^{-2} - x^{-3}$

- $4x^{\frac{1}{2}} - (4x)^{-\frac{1}{2}}$

# Factorising

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Factorise

- $4x^{-2} - x^{-1} - 3$

- $(2x)^{-2} - (4x)^{-1} - 3$

# Simultaneous eqns

"Solve these simultaneous equations" at GCSE

- $2x - 3y = 5$
  $3x + 4y = 16$

- $x^2 + y^2 = 40$
  $x - y = 4$

# Simultaneous eqns

"Solve these simultaneous equations" at A level

- $2x^2 - 3y^2 = 5$
  $3x^2 + 4y^2 = 16$

- $\sqrt{2}x - \sqrt{3}y = 9$
  $4x + \sqrt{6}y = 9\sqrt{2}$

- $T\cos 40° - S\cos 70° = 20$
  $T\sin 40° + S\sin 70° = 49$

# Simultaneous eqns

Solve

- $x^2 + y^2 = 13$
  $2x + y = 4$

- $x^2 + y^2 = 13$
  $3x + 2y = 5$

- $3x^2 + 2y^2 = 35$
  $2x - 3y = 12$

# Simultaneous eqns

Solve

- $x^2 + xy + y^2 = 7$
  $3x + 2y = 5$

- $x^2 + y^2 = 13$
  $x^4 + y^4 = 97$

# Algebraic fractions

Simplify

- $2x^{-3} + 3x^{-3}$

- $2x^{-3} + 3x^{-2}$

- $(2x)^{-3} + (3x)^{-2}$

# Algebraic fractions

Simplify

- $(x+1)^{\frac{1}{2}} - (x+1)^{-\frac{1}{2}}$

- $(9x+9)^{\frac{3}{2}} - x(4x+4)^{\frac{1}{2}}$

# Algebraic fractions

Solve

- $2x(x^4+1)^{-0.5} - 2x^3(x^2+1)(x^4+1)^{-1.5} = 0$

# Algebraic fractions

Simplify

- $\frac{3x^2 - 4x^5}{x^4}$

- $\frac{4x + 5}{3x}$

- $\frac{2x^2\sqrt{x} + 9x}{\sqrt{3x^3}}$

# Algebraic fractions

Simplify

- $\frac{x^2}{x+1}$

- $\frac{x^2+1}{x+1}$

- $\frac{x^2+3x}{x+1}$

# Indices …

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Simplify

- $\sqrt{36x^{36}}$
- $\sqrt[3]{216y^{216}}$
- $(3x)^{-2} \times 2x^{-3}$
- $3x^{-2} \div (2x)^{-3}$

# Indices …

If $2^a = 9$, evaluate

- $2^{a+2}$
- $2^{-a}$
- $2^{\frac{a}{2}}$
- $4^{a-1}$

# … & logarithms

If $a = \log_2 9$, simplify

- $2^a$
- $4^{-a}$

Express in terms of $a$

- $\log_2 18$
- $\log_2 \frac{8}{9}$
- $\log_4 3$

# … & logarithms

If $a = \frac{1}{2}\log_2 6$, simplify

- $12 \div 2^a$
- $\frac{4a+1}{4a-1}$
- $\frac{4^a+4^{-a}}{4^a-4^{-a}}$

# … & logarithms

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If $a = \sqrt{e}$ and $b = -\ln 4$, simplify

- $a \ln a$
- $\ln\left(\frac{1}{a^2}\right) + \frac{1}{(\ln a)^2}$
- $\frac{a^2}{1-\ln a}$
- $b - \ln 2$
- $e^{\frac{1}{2}b}$

# How we learn

When a new concept is encountered, in order for learning to take place, one of two things must happen:

either

- the new idea is connected to and incorporated into the learner's existing framework of knowledge (**assimilation**)

or

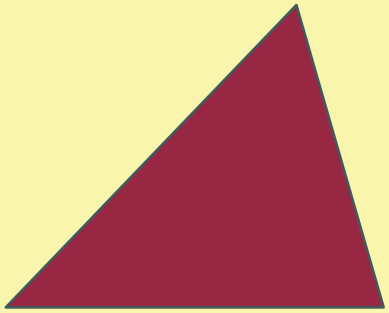
- existing knowledge structures must be extended / reorganised (**accommodation**).

Richard Skemp, 1976 & 1986

# How we learn

Area = 20cm$^2$
Base = 5cm
Height = ?

# How we learn

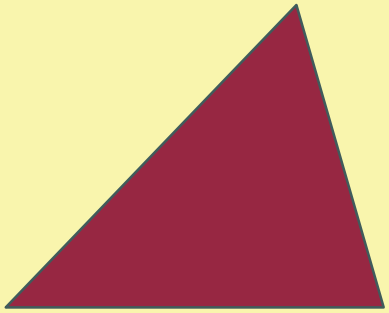


Area = 10cm$^2$
Base = 5cm
Height = ?

Area = 20cm$^2$
Base = 5cm
Height = ?

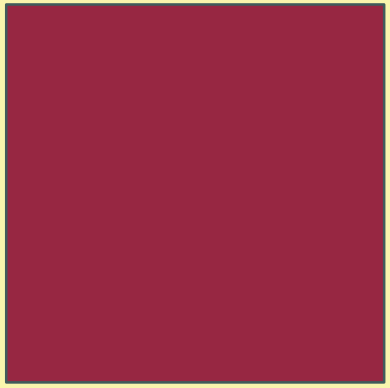
**Assimilation**

# How we learn



Area = 10cm$^2$
Base = 5cm
Height = ?

Area = 20cm$^2$
Base = 5cm
Height = ?

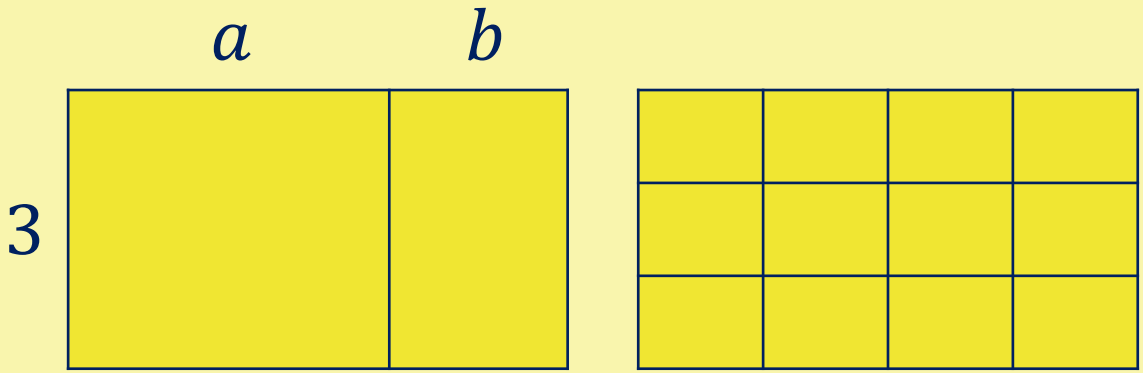


Area = 25cm$^2$
Height = ?

**Assimilation**

**Accommodation**

# How we learn






$3 \times a$

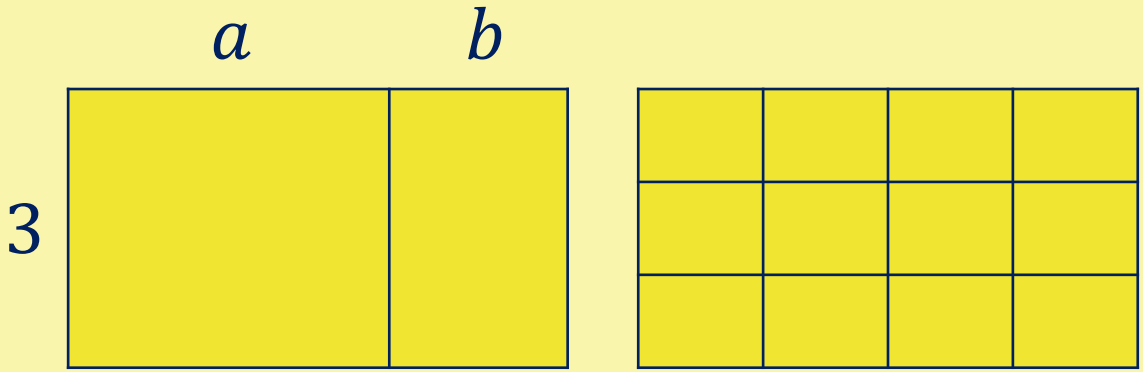
$3(a + b)$

$3 \times 4$

**Assimilation**

**Accommodation**

# How we learn




$3 \times a$
$3(a+b)$

$3 \times 4$

$\sin(a+b)$
$\ln(a+b)$

**Assimilation**

**Accommodation**

# How we learn

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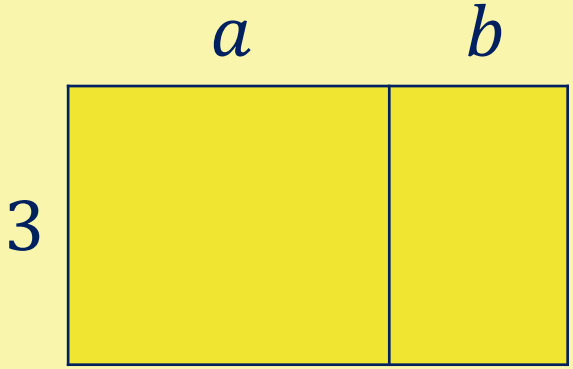
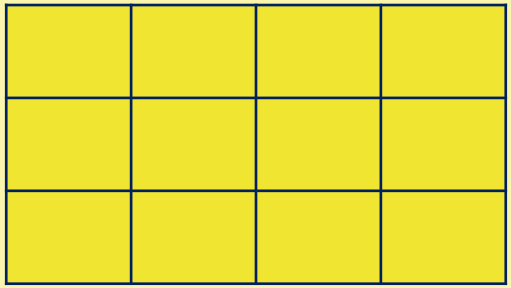
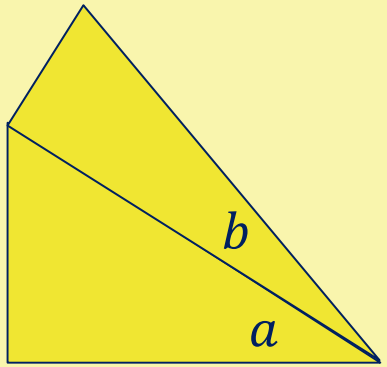



$3 \times a$

$3(a+b)$



$3 \times 4$




$\sin(a+b)$

$\ln(a+b)$

**Assimilation**

**Accommodation**

# How we learn

So, for teaching to result in meaningful learning, we need to know students' prior knowledge.

Vygotsky's idea of **zone of proximal development** (ZPD) is helpful here. ZPD is "the distance between where the student is on their own, and where they can get to with the help of a more knowledgeable other … **learning things that are just beyond where their current understanding is**".

Vygotsky, 1962

# Algebra-tisation

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8 Without using a calculator, work out the exact values of

a) $\frac{101^2-1}{102}$ b) $\frac{101^2-1}{25}$ c) $\frac{101^4-101^2}{101^3-101^2}$

# Algebra-tisation

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| | |
|---|---|
| 9 | Person X donates 10% of their salary each month to charity. Person Y donates 20% of their salary each month to charity. The amount Y donates is 50% more than the amount X donates. What percentage increase in Y's salary will make it equal to X's salary? |

# Algebra-tisation

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10 A and B share a whole number of £. They each get a whole number of £, and then A gives B £10. The ratio of their shares is now 3 : 2. How much might they each now have?

# In conclusion

- “Unfinished business” leads to “trouble ahead”
- General tactics & Specific strategies
- Desirable difficulty, productive struggle
- Get the maths from them
- Accommodate don’t assimilate

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What might we do first?

What might we do next?

Agree or challenge?

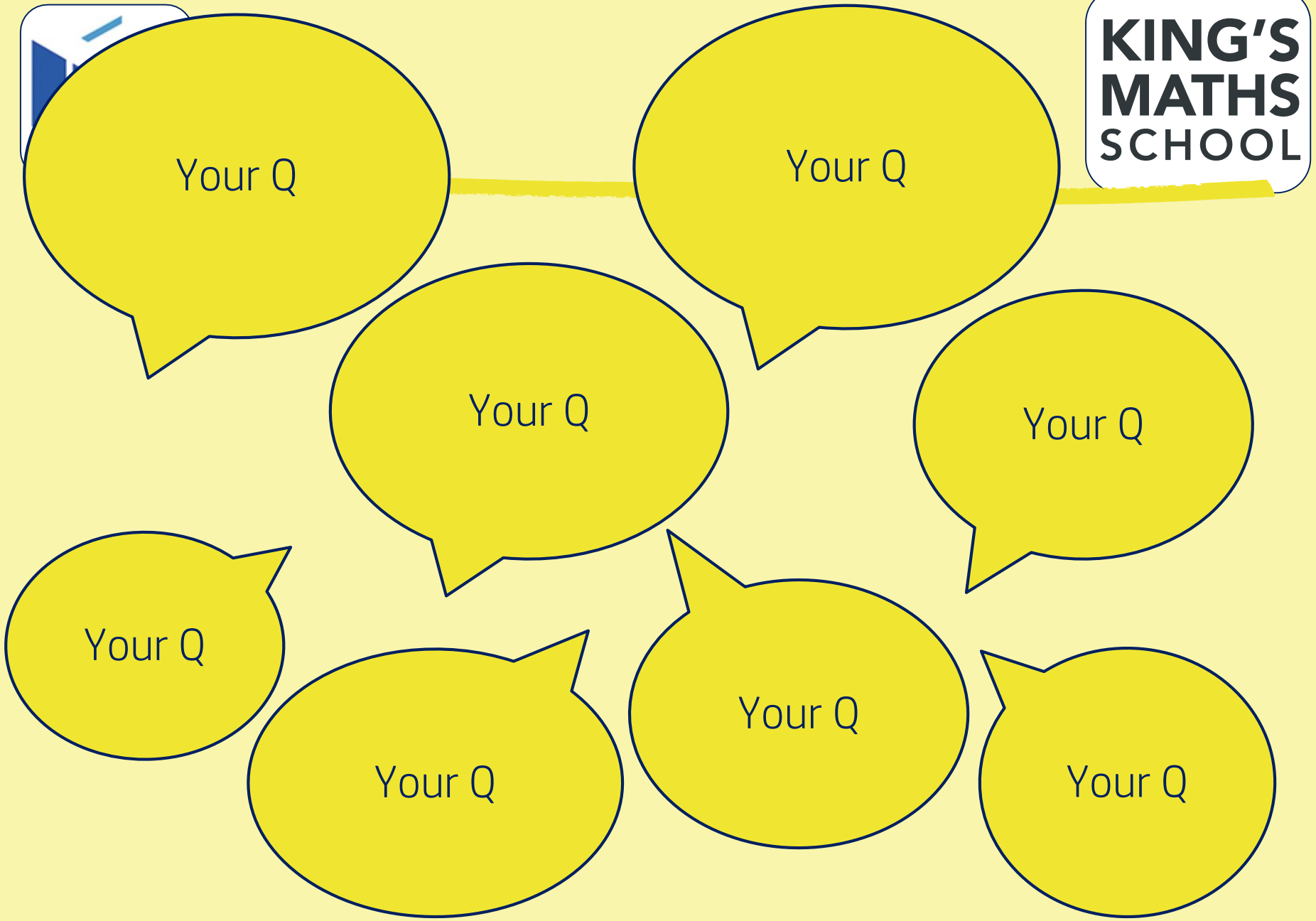
Why did we … ?

Are you sure?

What's the same, what's different

What do you notice?

Why didn't we … ?


Your Q
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Your Q


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# Thank you

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#TeachersAlwaysLearning

#LearningTogether

@kingsmathschool