Hounslow Education Partnership

KING'S MATHS SCHOOL

Start as they need to go on

Developing graph skills for A level success

Please

- change your Zoom name to "Full name, School"
- keep your video on but microphone off
- Have a paper and pencil, or something more modern!, to hand
- Tell us in the Chat: "My favourite chocolate is ..."

My objectives

To suggest

- topics
- examples
- rationale

to support and inform your construction of a Y12 “Core Ideas in Graphs” curriculum thread that will prepare your students well for Y13 success.

ALL students must

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REASON MATHEMATICALLY by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

ALL students must

become **FLUENT** in the fundamentals of mathematics, including through **varied and frequent practice with increasingly complex problems** over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

[illegible] justification or proof using mathematical language.

ALL students must

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become **FLUENT** in the
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language.

SOLVE PROBLEMS by applying their mathematics to a **variety of routine and non-routine problems** with increasing sophistication, including breaking down problems into a series of simpler steps and **persevering in seeking solutions**.

Routine and non-routine

- Completing the Square
- Sketching graphs
- Intersecting graphs
- Inequalities
- Transforming graphs
- (and some lovely circle stuff ☐)

In conclusion

- “Unfinished business” leads to “trouble ahead”
- General tactics & Specific strategies
- Desirable difficulty, productive struggle
- Get the maths from them
- Accommodate don’t assimilate

In conclusion

- "Un[illegible]ed b[illegible]" [illegible]ble ahead
- Ge[illegible]
- Desirable [illegible]ggle
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Graphs are representations of functions

In conclusion

- "U[illegible]ed b[illegible]" [illegible]ble ahead
- G[illegible]
- Desirable [illegible]ggle
- Get the maths [illegible]m the[illegible]
- **Accommodate don't assimilate**

Graphs are representations of functions

Completing the square

What is the minimum value of these functions?

- $x: \rightarrow x^2 - 8x - 2$
- $x: \rightarrow x^4 - 8x^2 - 2$
- $x: \rightarrow x^4 + 8x^2 - 2$
- $x: \rightarrow x^6 + 8x^3 - 2$
- $x: \rightarrow \sin^2 x + 8 \sin x - 2$

Completing the square

Hence sketch the graphs representing the functions

- $x: \rightarrow x^2 - 8x - 2$

- $x: \rightarrow x^4 - 8x^2 - 2$

- $x: \rightarrow x^4 + 8x^2 - 2$

Not-completing the …

… factorisation: what are the coordinates of the vertex of the graph representing the functions

- $x: \rightarrow x^2 - 8x - 2$
- $x: \rightarrow 2x^2 - x - 1$
- $x: \rightarrow -x^2 + 3x + 3$

"Choose wisely"

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Sketch the graphs representing the following functions. State the coordinates of the y-intercept and choose whether to state the coordinates of the x-intercept(s) or the vertex:

- $x: \rightarrow x^2 + 6x + 9$
- $x: \rightarrow x^2 + 6x + 7$
- $x: \rightarrow x^2 + 7x + 6$

"Choose wisely"

KING'S MATHS SCHOOL

Sketch the graphs representing the following functions. State the coordinates of the y-intercept and choose whether to state the coordinates of the x-intercept(s) or the vertex:

- $x: \rightarrow (x+3)^2 + 9$
- $x: \rightarrow 9 - (x+3)^2$
- $x: \rightarrow 2(x+3)^2 - 9$

"Thinking backwards"

Give the equation of the quadratic graphs with the properties stated:

- The graph crosses the x-axis at (2, 0) and (–3, 0). It has y-intercept at (0, –12).
- The graph crosses the x-axis at (1, 0) and (4, 0). It has y-intercept at (0, –4).
- The graph touches the x-axis at (–2, 0). It has y-intercept at (0, 12).

"Thinking backwards"

KING'S MATHS SCHOOL

Give the equation of the quadratic graphs with the properties stated:

- The graph has its vertex at (–3, 5). It has y-intercept at (0, 14).
- The graph has its vertex at (1, –4). It has y-intercept at (0, –2).
- The graph has its vertex at (2, 7). It has y-intercept at (0, 3).

"Thinking backwards"

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Equation of line	y-intercept	gradient	x-intercept
$y = 4x - 7$	$(0, -7)$	4	$\left(1\frac{3}{4}, 0\right)$
$5y = 2x - 4$			
$4y + x + 6 = 0$			
	$(0, -3)$	2	
	$(0, 3)$	0	
	$(0, -4)$	$\frac{2}{3}$	
	$(0, 6)$		$(2, 0)$
		$-\frac{4}{5}$	$(-2, 0)$

"Thinking backwards"

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4 A line has gradient $-\frac{1}{2}$. $(5, 2)$ is a point on the line.

Complete the coordinates of these points which are also on the same line:

a) $(9, \_\_)$ b) $(-4, \_\_)$ c) $(\_\_, -2)$

"Thinking sideways"

Work out the area of the triangle with vertices at

- (−2, 0), (3, 5) and (4, −2).

Cubic graphs

State the coordinates of the y-intercept and work out the x-intercept(s) of the graphs representing the following functions:

- $x: \rightarrow x^3 - 2x^2 - 11x + 12$
- $x: \rightarrow x^3 - 14x - 8$
- $x: \rightarrow x^3 - 8x^2 + 20x - 16$
- $x: \rightarrow x^3 - 8x^2 + 21x - 20$
- $x: \rightarrow x^3 - 6x^2 + 11x - 12$
- $x: \rightarrow x^3 - 6x^2 + 12x - 16$

Cubic graphs

Determine the coordinates

- where $y = -8$ on $y = x^3 - 6x^2 + 12x - 16$

- which satisfy both $y = x^3 - 6x^2 + 11x - 12$ and $y = 2x - 8$

- which satisfy both $y = x^3 - 6x^2 + 11x - 12$ and $y = -x - 4$

Inequalities

Sketch the graph representing the function:

- $x: \rightarrow 4x + 6$

Work out

- the exact value of x that satisfies $4x + 6 = 11$
- the range(s) of values of x that satisfy
  - $4x + 6 < 11$
  - $4x + 6 > 2$

Inequalities

Sketch the graph representing the function:

- $x: \rightarrow x^2 + 4x + 6$

Work out

- the exact values of x that satisfy $x^2 + 4x + 6 = 3$
- the range(s) of values of x that satisfy
  - $x^2 + 4x + 6 > 6$
  - $x^2 + 4x + 6 < 11$
  - $x^2 + 4x + 6 > 2$

Inequalities

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Sketch on the same axes the graphs representing the functions:

- $x: \rightarrow x^2$
- $x: \rightarrow 9$

Write down

- the exact values of x that satisfy $x^2 = 9$
- the range(s) of values of x that satisfy $x^2 > 9$

Agree or challenge: "$x^2 > 9 \Rightarrow x > 3$ or $x > -3$"

Inequalities

Sketch on the same axes the graphs representing the functions:

- $x: \rightarrow x^2$
- $x: \rightarrow 3x$

Write down

- the exact values of x that satisfy $x^2 = 3x$
- the range(s) of values of x that satisfy $x^2 > 3x$

Agree or challenge: "$x^2 > 3x \Rightarrow x > 3$"

Inequalities

Sketch on the same axes the graphs representing the functions:

- $x: \rightarrow \frac{4}{x}$
- $x: \rightarrow x$

Write down

- the exact values of x that satisfy $\frac{4}{x} = x$
- the range(s) of values of x that satisfy $\frac{4}{x} < x$

Agree or challenge: "$\frac{4}{x} < x \Rightarrow x^2 > 4 \Rightarrow x > 2$ or $x < -2$"

Inequalities

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Sketch on the same axes the graphs representing the functions:

- $x: \rightarrow 0.5x^3$
- $x: \rightarrow x^2$

Write down

- the exact values of x that satisfy $x^2 = 0.5x^3$
- the range(s) of values of x that satisfy $x^2 > 0.5x^3$

Agree or challenge: "$x^2 > 0.5x^3 \Rightarrow 1 > 0.5x \Rightarrow x < 2$"

Transforming graphs

Sketch the graphs representing the functions:

- $x: \to x^2$ and $x: \to (x-2)^2$
- $x: \to x^2$ and $x: \to (x-2)^2 + 3$
- $x: \to x^2$ and $x: \to x^2 - 4x + 1$

Transforming graphs

Sketch the graphs representing the functions:

- $x: \to x^2$ and $x: \to x^2 - 4x + 1$
- $x: \to x^2$ and $x: \to -x^2 + 4x - 1$
- $x: \to x^2$ and $x: \to x^2 + 4x + 1$
- $x: \to x^2$ and $x: \to 4x^2 - 8x + 1$

Transforming graphs

Sketch the graphs representing the functions:

- $x: \to x^3$ and $x: \to (2x)^3 + 1$
- $x: \to x^3$ and $x: \to (-2x)^3 + 1$
- $x: \to x^3$ and $x: \to 2x^3 + 1$

Transforming graphs

Sketch the graphs representing the functions:

- $x: \to \frac{1}{x}$ and $x: \to \frac{1}{x} + 2$
- $x: \to \frac{1}{x}$ and $x: \to \frac{1}{x+2} + 2$
- $x: \to \frac{1}{x}$ and $x: \to \frac{2}{x+2} + 1$
- $x: \to \frac{1}{x}$ and $x: \to \frac{3x+4}{x+2}$

Transforming graphs

Sketch the graph representing the function:

- $x: \rightarrow f(x)$ where $f(x) = 6x - x^2$

Now express each of these in terms of f, and hence sketch the graph of each function:

- $x: \rightarrow 6(x + 2) - (x + 2)^2$
- $x: \rightarrow -6x - x^2$
- $x: \rightarrow 5 + 12x - 4x^2$

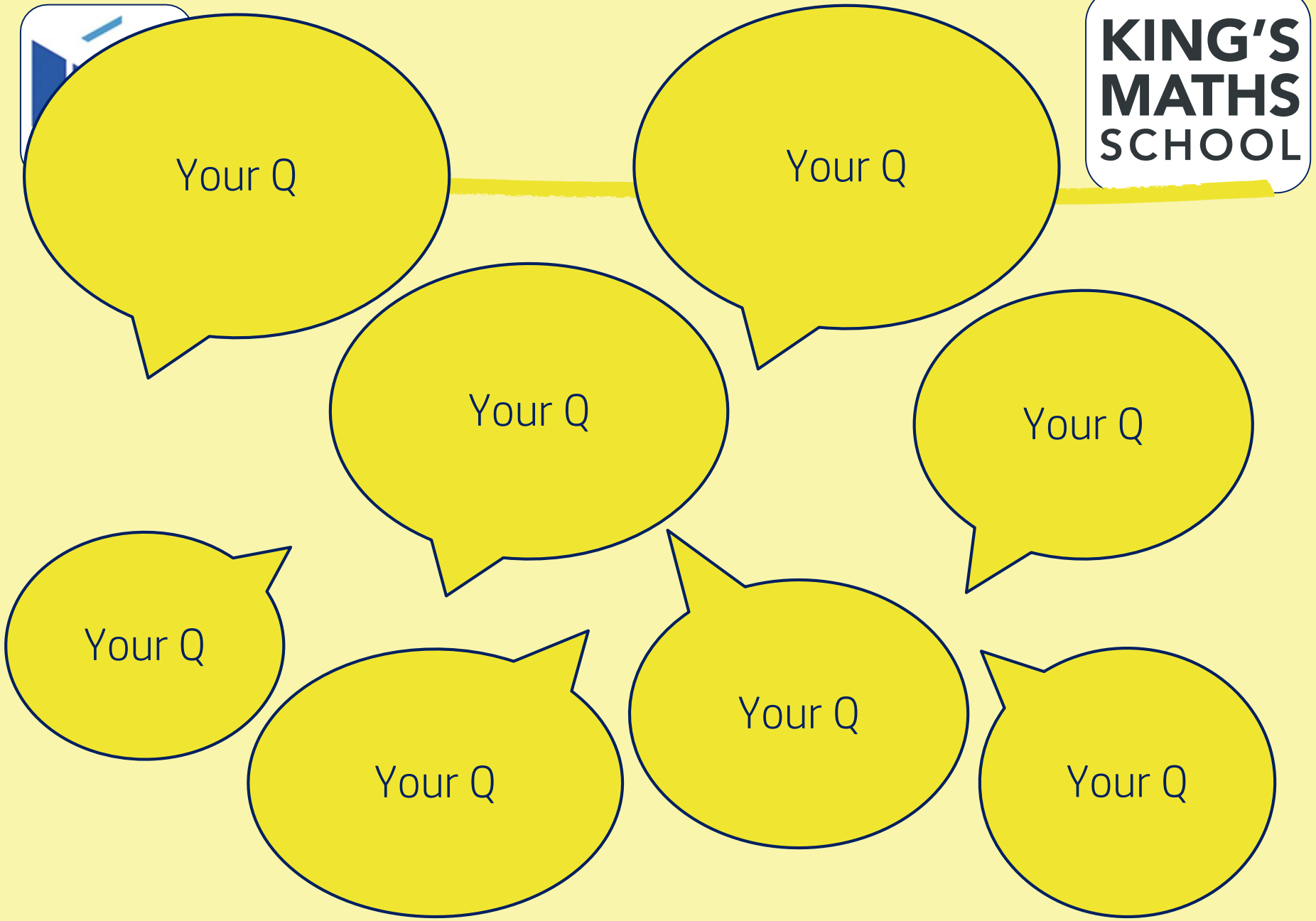
Transforming graphs

Sketch the graph representing the function:

- $x: \rightarrow f(x)$ where $f(x) = x(x-4)$

Now express each of these in terms of f, and hence sketch the graph of each function:

- $x: \rightarrow x^2 - 2x - 3$
- $x: \rightarrow 16x(x-1)$
- $x: \rightarrow (x-8)(x+4)$
- $x: \rightarrow (x-8)(x-4)$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Intersecting circles

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9 Do the two circles

- one with equation $x^2 + y^2 = 36$
- one with centre $(6, 8)$ and radius 4

intersect at two different points, touch at one point, or not intersect at all?

Q10 (prep qns)

Intersecting circles

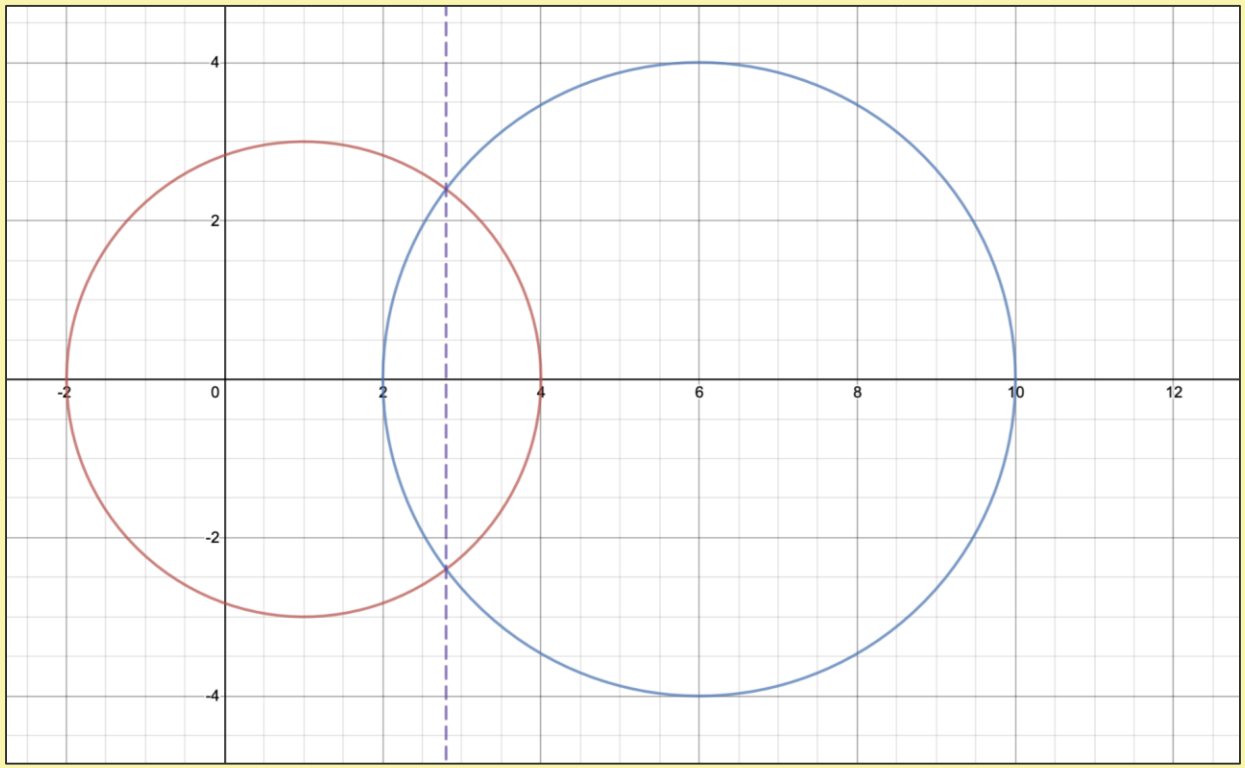
Find the intersection(s) of

- $x^2 + y^2 - 2x - 8 = 0$
- $x^2 + y^2 - 12x + 20 = 0$

Intersecting circles

Find the intersection(s) of

- $x^2 + y^2 - 2x - 8 = 0$
- $x^2 + y^2 - 12x + 20 = 0$



Intersecting circles

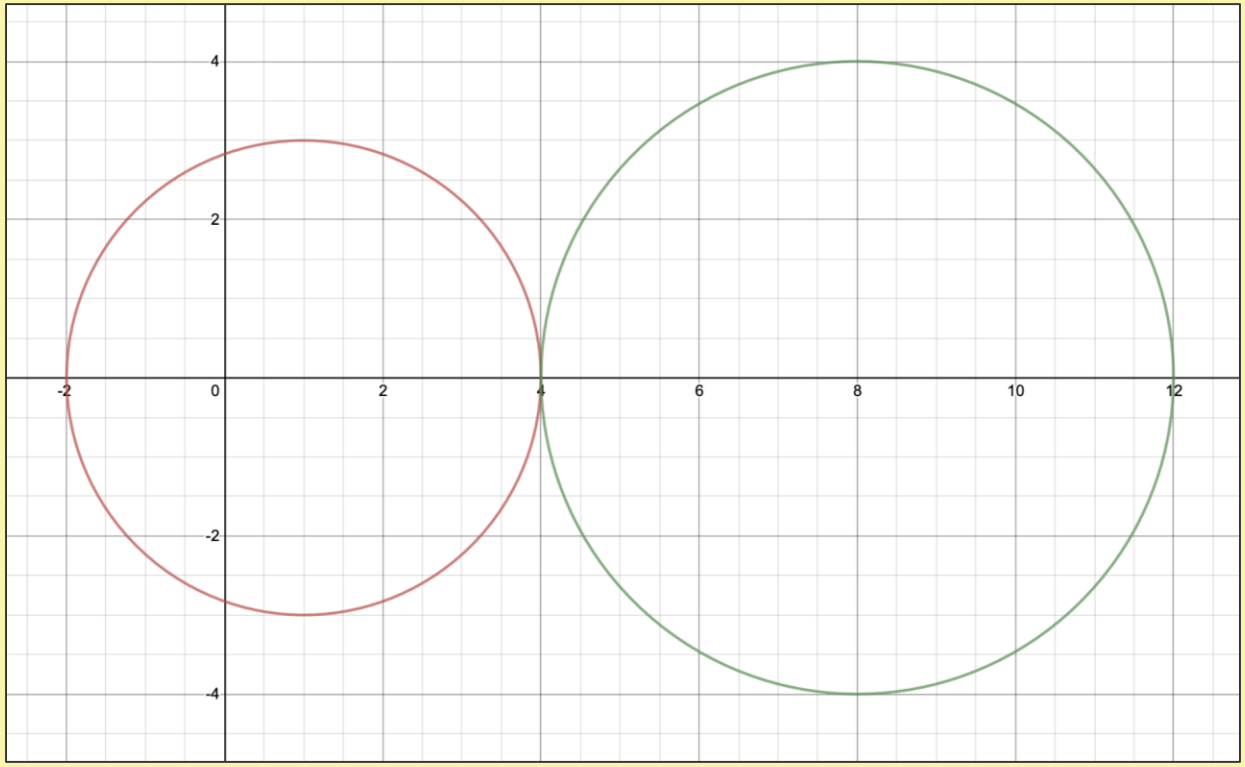
Find the intersection(s) of

- $x^2 + y^2 - 2x - 8 = 0$
- $x^2 + y^2 - 16x + 48 = 0$

Intersecting circles

Find the intersection(s) of

- $x^2 + y^2 - 2x - 8 = 0$
- $x^2 + y^2 - 16x + 48 = 0$



Intersecting circles

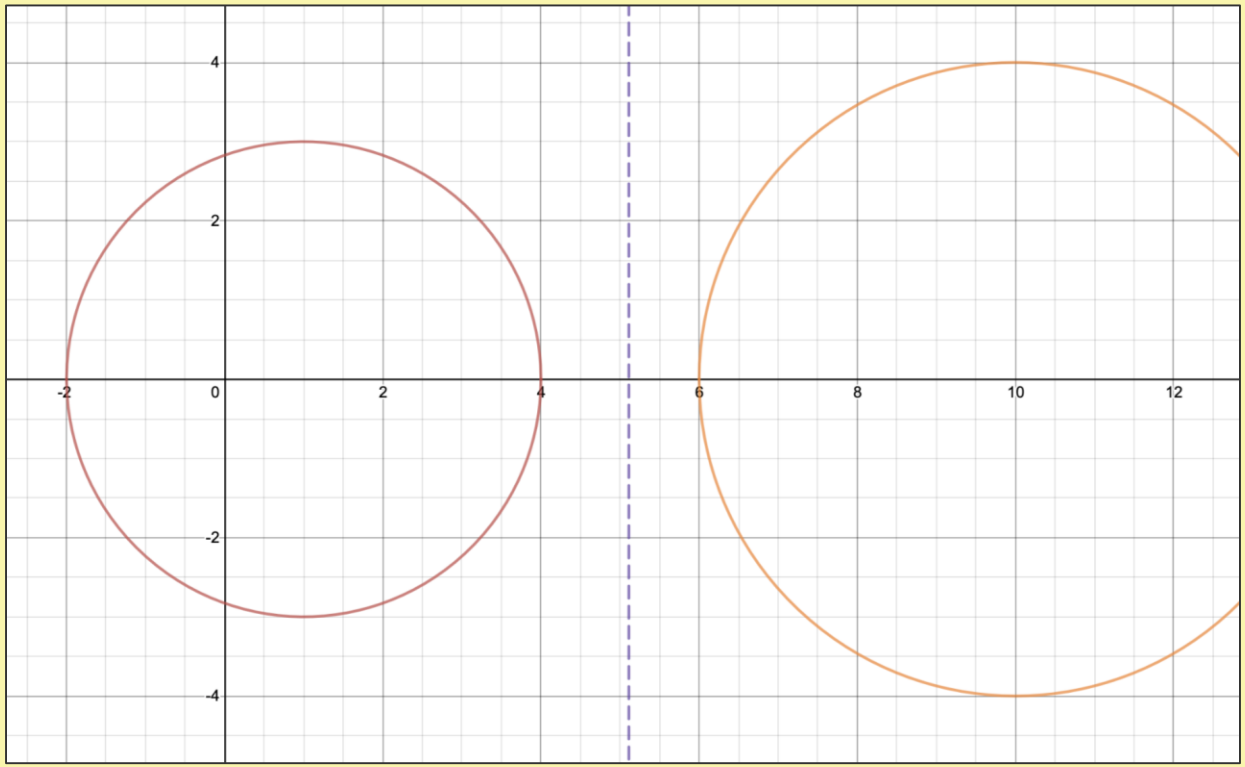
Find the intersection(s) of

- $x^2 + y^2 - 2x - 8 = 0$
- $x^2 + y^2 - 20x + 84 = 0$

Intersecting circles

Find the intersection(s) of

- $x^2 + y^2 - 2x - 8 = 0$
- $x^2 + y^2 - 20x + 84 = 0$

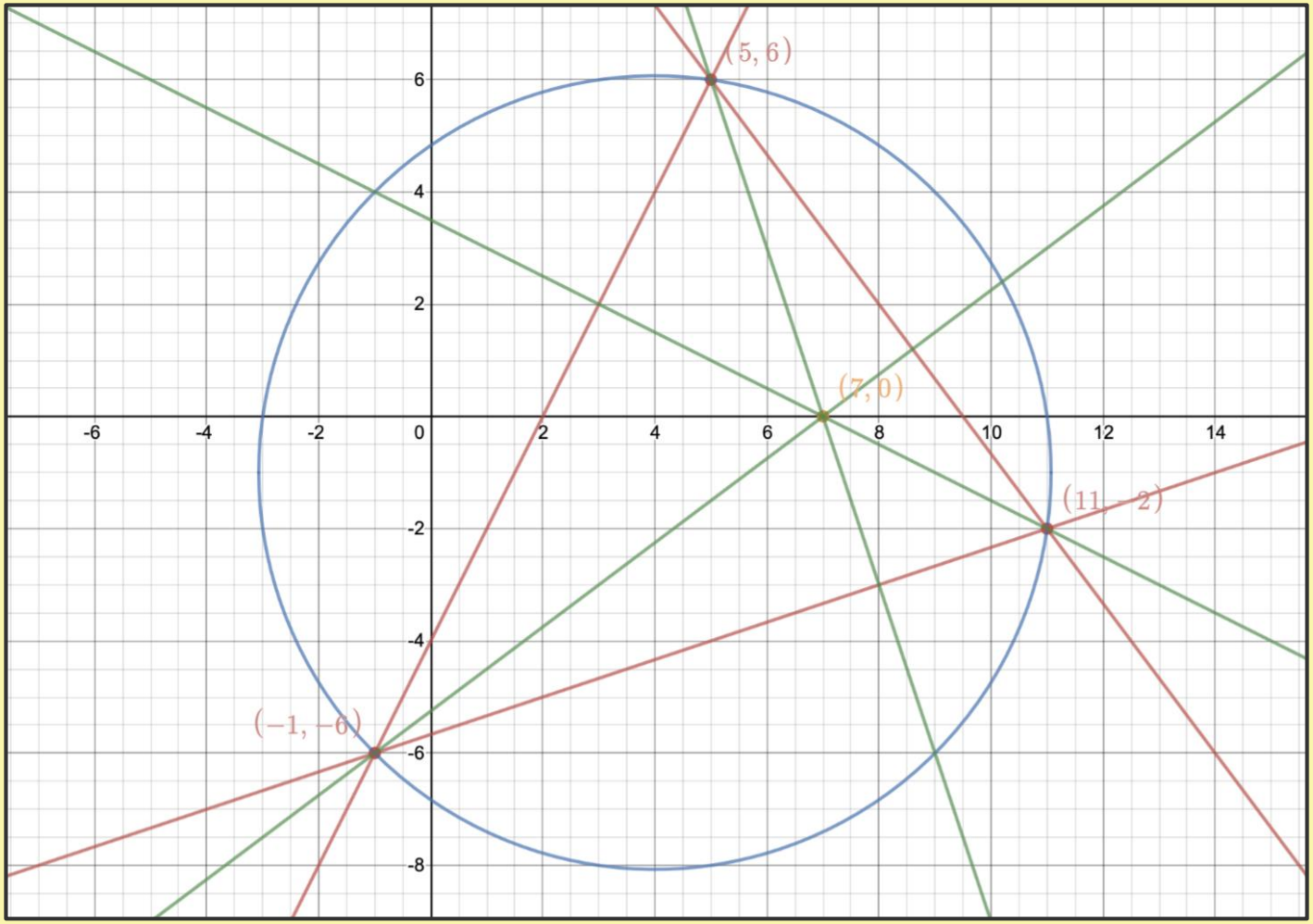


The altitudes of a triangle

A triangle has vertices at (-1, -6), (5, 6) and (11, -2)

Verify that the three altitudes are **concurrent** (at the **orthocentre**).

The altitudes



In conclusion

- “Unfinished business” leads to “trouble ahead”
- General tactics & Specific strategies
- Desirable difficulty, productive struggle
- Get the maths from them
- Accommodate don’t assimilate

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What might we do first?

What might we do next?

Agree or challenge?

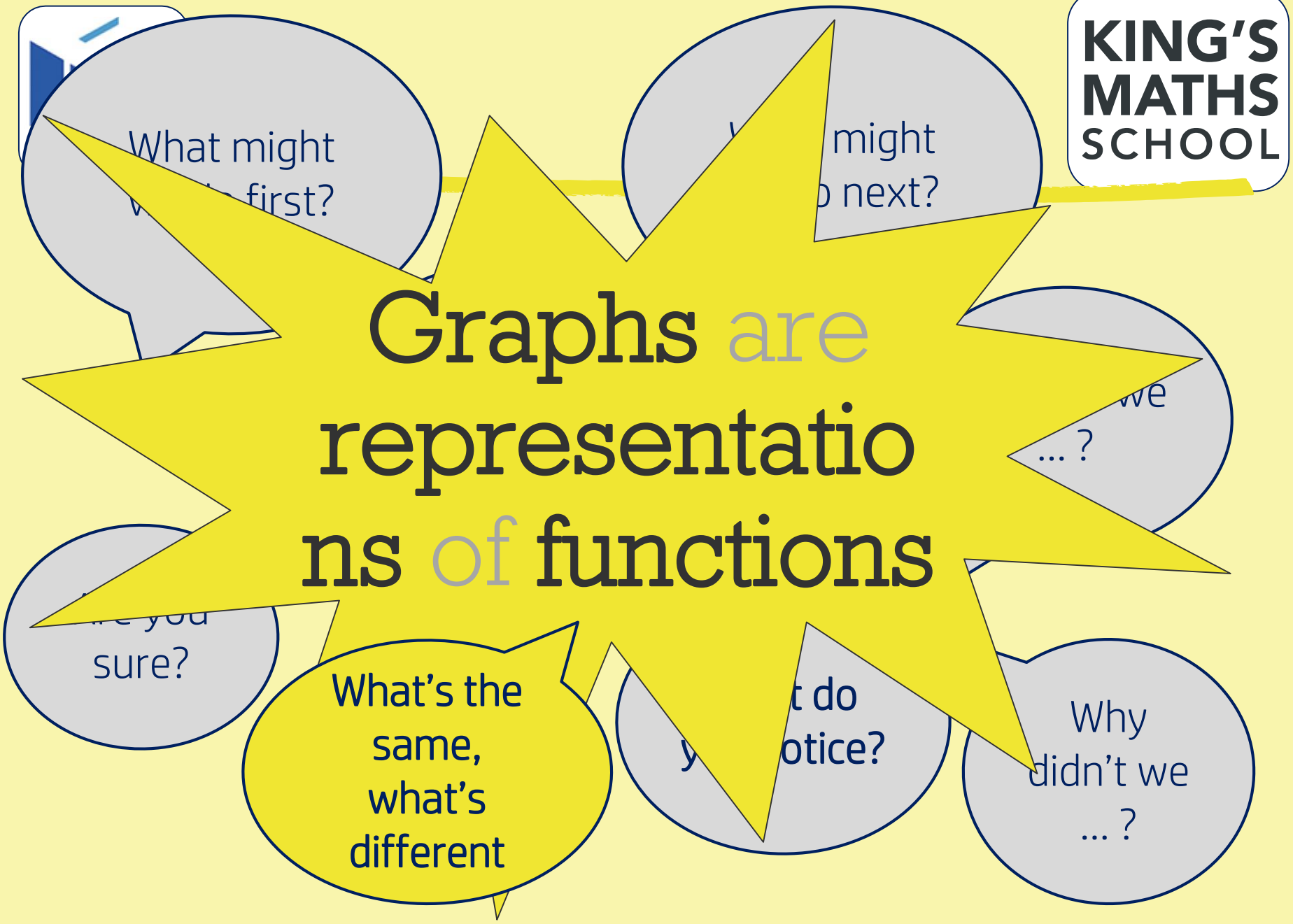
Why did we … ?

Are you sure?

What's the same, what's different

What do you notice?

Why didn't we … ?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Graphs are representations of functions

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Thank you

Robert Wilne

robert.wilne@kcl.ac.uk

#TeachersAlwaysLearning

#LearningTogether

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